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STRUCTURE-BORNE SOUND POWER AND SOURCE CHARACTERISATION IN MULTI-POINT-CONNECTED SYSTEMS, PART 1: CASE STUDIES FOR ASSUMED FORCE DISTRIBUTIONS

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At the design stage of many projects, the engineer is often asked to make a prediction of any resultant sound levels. To achieve this goal, the analysis needs to account for all the excitation sources and their interaction with all the transmission paths. Whilst for air-borne sound a source–path–receiver model is often employed with success, inherent physical problems have, to date, prevented a similar approach being adopted for structure-borne sound. In this paper the problems are reexamined with particular reference to the characterization of machines as structure-borne sound sources. A novel approach proposed by Mondot and Petersson for a single point and component of motion [1] is developed to include multiple points.

1. INTRODUCTION

A problem fundamental to the control of structure-borne noise is the characterization of machines as vibrational sources. Strong dynamic coupling between a machine and supporting structure prevents source characterisation based upon the transmitted power. Whilst many alternative approaches have been proposed [2] these commonly assume either the receiver to be dynamically stable, e.g., to permit a constant force or velocity source assumption [3], or else assume a standard receiver structure i.e., the reception plate method [4]. Application of these ideas to the general case is not possible. A more promising approach proposed by Mondot and Petersson [1] characterizes a source based on its ability to deliver power. For a single point connection, with a single component of motion, the calculation assumes nothing about the receiver structure and can be applied to any case. A problem arises, however, in applying it to multi-point and multi-component connections. For these cases, the calculation can only be undertaken once an estimate of the force distribution amongst the connections has been made. The potential for a powerful noise control tool is therefore realizable providing the force distribution amongst the contact points can be adequately assumed.

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In this paper the viability of using various simple estimates of the force distribution is assessed by using idealized beam structures. This is a prelude to more detailed work, involving generalised structures, which will be reported in later papers.

2. THEORETICAL REVIEW

The structure-borne sound manifested in a body can be measured as either a force, F, or a velocity, V at any point n, and in any direction i: i.e., F_i^n or V_i^n. A quantization of the sound obtained by using just one of these measurements is however misleading. Consider for example the velocity resulting from a force where, if the structure is stiff, the magnitude of the velocity would clearly be less than if it were flexible. Further confusion arises because the differences in the units of rotation and translation make an assessment of the relative importance of different components of excitation difficult. There is a growing consensus therefore towards combining the two field measurements to calculate power [5].

If a vibrational source is attached to a receiver via a single, uni-directional point the transmitted power can be given by

$$Q = [(V_{sf})^2/2|Y_r + Y_s|^2]\,\mathrm{Re}\,\{Y_r\}, \tag{1}$$

where Y_S is the mobility of the source, Y_r the mobility of the receiver and V_{sf} is the r.m.s. free velocity of the source. The free velocity quantizes the response of the source to all of its internal vibration producing mechanisms and is the velocity measured whilst the machine is run under normal operating conditions but suspended in free space. Therefore, it represents the activity of the source.

Equation (1) is interesting for it illustrates why a structure-borne sound source characteristic cannot be extracted which is analogous to source strength in air-borne sound. In air-borne sound the source strength can be defined by the power transmitted at the surface–air interface. The formulation is analogous to equation (1) though with impedance (the inverse of mobility) and sound pressure the preferred variables. Although there are circumstances (such as a source surface in close proximity to a receiver surface) where the condition does not hold, the impedance of the receiver, i.e., air, can be assumed constant and independent of location. Hence the power involves only variables associated with the source and can be used directly to characterize a source.

In structure-borne sound there are many possible receivers and the receiver mobility will vary. The power is dependent upon both source and receiver and a characterization of source strength analogous to that for air-borne sound is therefore not possible. A different approach is required.

The problem has been addressed in a seminal paper by Mondot and Petersson [1]. Their approach is to introduce two functions; one, the source descriptor S, describes the ability of a source to yield power and the other, the coupling function Cf, is the proportion of this power which is manifested:

$$S = (V_{sf})^2/2Y_s^*, \qquad Cf = Y_s^* Y_r/|Y_s + Y_r|^2. \tag{2, 3}$$

The transmitted power is given by the real part of their product.

The formulation has several distinct properties. Firstly, the source descriptor is a unique function of the source in that it involves only source variables. Secondly, since the source descriptor has the units of power, different components of motion can be compared directly. Finally, for a single point uni-directional system, both the source descriptor and coupling function involve variables which are measurable prior to assembly and can therefore be used to make a prediction of the transmitted power.

If the source descriptor and coupling function can be applied to multi-point-connected systems a valuable tool for analysis will result.

2.1. SOURCE CHARACTERIZATION IN MULTI-POINT-CONNECTED SYSTEMS

Where the source and receiver are connected at many points and motion is possible in any of the six degrees of freedom (three translational and three rotational) the analysis becomes complicated because coupling between all points and all components of motion has to be taken into account. If both a source mobility matrix, $[Y_S]$, and a receiver mobility matrix, $[Y_r]$, are introduced together with a force ratio vector, $\{F_j^m/F_i^n\}$, the transmitted power can be written as

$$Q_i^n = \frac{(V_{sf_i}^n)^2}{2|[Y_s]\{F_j^m/F_i^n\} + [Y_r]\{F_j^m/F_i^n\}|^2} \mathrm{Re}\,\{[Y_r]\{F_j^m/F_i^n\}\}. \tag{4}$$

The equation reveals the problems and complications associated with the analysis of multi-point-connected systems. Four are key.

(i) The matrices involved are large. For N points, both the source and receiver mobility matrices will be of size $6N \times 6N$: that is, 576 elements for even a simple four point system. Even if reciprocity is invoked the amount of data is still considerable. The problem is compounded since the matrix elements are complex and usually complicated functions of frequency.

(ii) There are difficulties in assessing the relative significance of source mobility compared with that of the receiver mobility: i.e., with so many mobility and force terms it is difficult to determine whether a constant force or constant velocity source idealization can be assumed [3].

(iii) There are difficulties in assessing the relative importance of the coupling between different directions and points with regard to the power component under consideration: i.e., the most important transmission path/s.

(iv) Knowledge of the forces, or more specifically the force ratios, acting at the interface is required.

To help address these problems the effective point mobility can be introduced [6]. The concept is based upon the premise that any point in a multi-point-connected system can be considered individually if the effects on that point of all other points and components of motion are taken into account. It can be expressed as [7]

$$Y_{ii}^{nn\Sigma} = Y_{ii}^{nn} + \sum_{m=1, m\neq n}^{N} Y_{ii}^{nm}\frac{F_i^m}{F_i^n} + \sum_{j=1, j\neq i}^{6} Y_{ij}^{mn}\frac{F_j^n}{F_i^n} + \sum_{j-1, j\neq i}^{6} Y_{ij}^{mn}\frac{F_j^m}{F_i^n}: \tag{5}$$

i.e., the product of the mobility matrix and the force ratio vector in equation (4) is expressed as a linear combination of terms. As written, the first term is the direct contribution, the second (sum) is the contribution from the other points in the direction of motion being considered, the third is the contribution from the other directions at the point being considered and the fourth is the contribution from the other points in directions other than that being considered. Since the effective point mobility is a single figure, its introduction addresses problems (i), (ii) and (iii). Problem (iv) does however still remain: i.e., knowledge of the forces.

For multi-point-connected systems, the effective point mobility is analogous to the point mobility. With reference to equations (2) and (3) the source descriptor and coupling function for a point n and in direction i for a multi-point-connected system are therefore given by

$$S_i^n = (V_{sf_i}^n)^2/2Y_{sii}^{nn\Sigma*}, \qquad Cf_i^n = Y_{sii}^{nn\Sigma*}Y_{rii}^{nn\Sigma*}/|Y_{sii}^{nn\Sigma} + Y_{rii}^{nn\Sigma}|^2, \tag{6, 7}$$

respectively. The transmitted power Q_i^n at a point and a direction is given by the real part of the product and the total transmitted power by the sum of all Q_i^n.

Thus far, the formulation of the source descriptor and coupling function has been analytical and the problem of practical application remains. This centres upon the construction of the effective point mobilities which in turn centres upon obtaining all of the mobility and force ratio terms. For the surface descriptor to be a function of only source parameters, the effective point mobility $Y_{S_{ii}}^{nn\Sigma}$ must be independent of the receiver. This requires that both the source mobilities and force ratios must also be independent.

The mobilities are dependent only upon the source structure and although acquisition is laborious they can, through either analytical techniques or measurement, be secured independently of the receiver. The force ratios however are inherently dependent upon both the source and receiver structures. If the source descriptor is to be a function of only source parameters the force ratios must therefore be predicted or assumed in some manner. Hence, before the source descriptor can be used an understanding of the force ratios is required. At the very least, the sensitivity of the source descriptor to estimates of the force ratios should be investigated. This is at the core of this study. It is important to note though that it is the force ratios, i.e., the force distribution, rather than the absolute forces which need to be predicted.

2.2. SIMPLE FORCE RATIO ASSUMPTIONS

Present understanding of the force ratios in a source-receiver system is very limited and it is only through the formulation of the effective point mobility that they have received any consideration at all. In the theoretical work of Mondot, Petersson and Gibbs [7–9] the translational force ratios have simply been assumed to have unity magnitude and zero phase. For force ratios with differing components of motion similarly arbitrary real and positive values have been used [7]. Upon assuming translational motion only, estimates of the effective point mobility have therefore been given by

$$Y_{ii}^{nn\Sigma} = Y_{ii}^{nn} + \sum_{m=1, m \neq n}^{N} Y_{ii}^{nm}. \tag{8}$$

The advantages of the assumption are clear; it is simple and does not rely upon knowledge of the receiver system. The independence of the source descriptor formulation is therefore maintained.

Alternatively, the contributions from the N contact points can be assumed to cancel through superposition (upon assuming that the system is linear); i.e., each mobility and force ratio has different phase. In this case the effective point mobility will simply reduce to the point mobility:

$$Y_n^{nn\Sigma} = Y_{ii}^{nn}. \tag{9}$$

This is the same as assuming that the contact points are uncoupled.

A third approach towards an estimate is to take into account both the excitation of the source and its structural characteristic. The simplest way is to assume that the excitation is the free velocity of the contact points and that the structure is characterized by the associated source point mobility. The force at a contact point is then given by

$$F_i^n = V_{sf_i}^n / Y_{Sii}^{nn}. \tag{10}$$

The omission of the receiver condition prescribes that this estimate is most applicable to an ideal force source condition.

To improve upon the third estimate, some account would have to be given to the coupling between contact points. This would lead towards complicated expressions for the forces (and subsequently the effective point mobilities) involving transfer mobility terms.

3. INITIAL INVESTIGATION

It is necessary to establish a criterion for the viability of estimating the force ratios. This could be the accuracy of an effective point mobility, source descriptor or coupling function. All are specific, however, to only one contact point and a comprehensive assessment based upon the contributions of all contact points and components of vibration would be a protracted task. A simplified investigation can instead be undertaken by selecting for the criterion the accuracy of the estimates of the total transmitted power. A clear implication of using this criterion is that no insight into the composite parameters is forthcoming. This is particularly limited for the investigation will profer nothing regarding the suitability of the source descriptor as an indicator of source strength. However, for much of engineering design the total transmitted power is of most interest and it can be argued that for the source descriptor/coupling function approach to be of use its prediction of the total transmitted power is the most important.

The response of a beam is not only relatively simple to model analytically but is also physically representative of many real structures. For an initial investigation, point connected beam source–receiver systems were therefore developed.

The Euler–Bernoulli wave equation was used to define the models [3],

$$EI\partial^4 W/\partial x^4 - \rho S \omega^2 W = 0, \tag{11}$$

where E is the Young's modulus, ρ the density, S the cross-sectional area, I the second moment of area of the cross-section, W the wave displacement and ω the frequency.

The general solution for a position x is

$$W(x) = A\,\mathrm{e}^{-\mathrm{i}kx} + B\,\mathrm{e}^{\mathrm{i}kx} + C\,\mathrm{e}^{-kx} + D\,\mathrm{e}^{kx} \tag{12}$$

where

$$k = (\omega^2 \rho S/EI)^{1/4}(1 - \mathrm{i}\eta), \tag{13}$$

is solved for the imposed boundary conditions.

The boundary conditions were based upon the source and receiver beams coupled together by rigid, massless connectors. With respect to Figure 1, conditions of continuity were imposed together with moment and force balance equations, which are respectively:

$$M_1 = M_3, \qquad M_2 = M_4 \qquad \text{and} \qquad F_1 + F_2 = F_3 + F_4. \tag{14, 15}$$

The boundary conditions allow power to be transmitted only by translational forces. Although the significance of moment induced power is recognized [10], the restriction is imposed because the thrust of the investigation is towards the realisation of the source descriptor/coupling function formulation rather than an analysis of the power in the

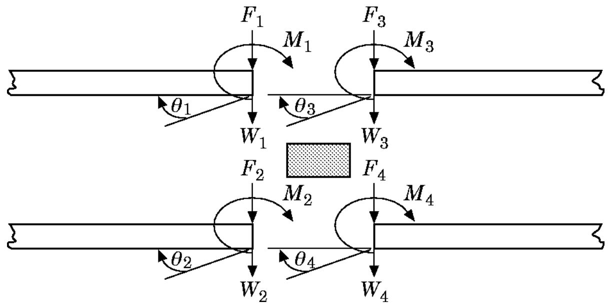

Figure 1. 'Exploded' diagram of a beam connection point.

systems. By assuming the power to be imparted by only translational forces the investigation could proceed with a much reduced set of mobility and force distribution functions, compared with the general case.

3.1. MOBILITIES

A reference system was designed; see Figure 2. This consists of a 5 mm thick finite source beam attached, via four connectors, to a 17 mm thick infinite receiver beam. Excitation of the system is by a force of unit magnitude and zero phase applied at an approximately central position. Both the source and receiver beam are assumed to have the material properties of steel with a loss factor of 0·001.

For the system, the magnitudes of the point and transfer mobilities associated with contact point 1 are shown in Figure 3(a) and the corresponding phases in Figure 3(b). In these and all subsequent figures, normalized wavelength denotes the number of wavelengths in the source beam. Two distinct frequency regions are exhibited. Firstly, a low frequency mass controlled region characterised by a 6 dB per octave decrease of magnitude and a frequency invariant phase difference of either $\pm\pi/2$. Secondly, a resonance controlled region in which both the magnitude and phase of the mobilities are highly variable with respect to frequency. The existence of the two regions is due to the source beam being finite and can be expected for any real structure.

Throughout the mass controlled region the magnitudes of the point mobility Y_S^{11} and the transfer mobilities Y_S^{12} and Y_S^{14} are approximately equal whilst that of Y_S^{13} is an order of magnitude less. In the resonance region it is difficult to establish a consistent relationship; whilst at certain frequencies the magnitudes are approximately equal it is more common that differences between them exist.

For the receiver the mobility magnitudes are shown in Figure 4. The receiver is infinite and the distinct mass and resonant controlled regions are therefore not seen. Instead, the magnitudes exhibit a monotonic decrease with increasing frequency. It is known [3] that the phase of the point mobility will be frequency invariant with a value of $-\pi/4$ but, due to the travelling wave, the phase of a transfer mobility will have a frequency dependence.

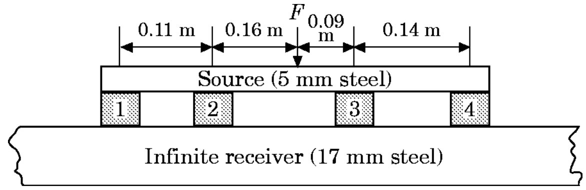

Figure 2. Reference system.

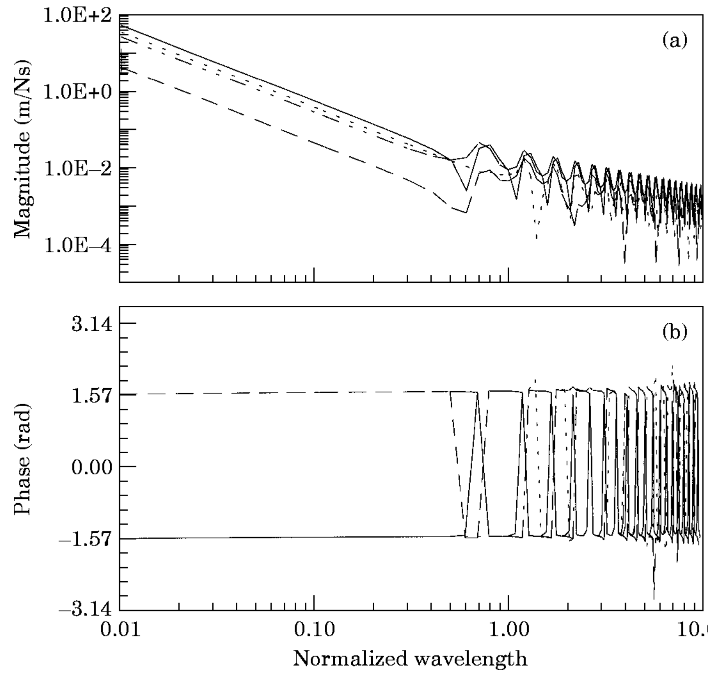

Figure 3. Source mobility (a) magnitude and (b) phase for reference system. —, Y_s^{11}; · · · ·, Y_s^{12}; – – –, Y_s^{13}; – · · –, Y_s^{14}.

At all frequencies the magnitudes of the receiver mobilities are approximately equal and, due to the greater thickness of the beam, are mostly at least 10^2, i.e., two decades, lower than those of the source.

3.2. FORCES

The magnitudes and phase differences (relative to the input force) of the four contact point forces are shown in Figures 5(a) and (b), respectively. As for the source mobilities two distinct regions are apparent: a low frequency region where the magnitudes are smooth in character and the phases discretized at either 0 or $\pm\pi$ and a high frequency region where both magnitude and phase are highly variable.

In the low frequency region the wavelengths are large compared with the bays (a bay defined as the distance between two adjacent connector points) and accordingly each bay responds as a rigid body; at very low frequencies the total source beam will respond as

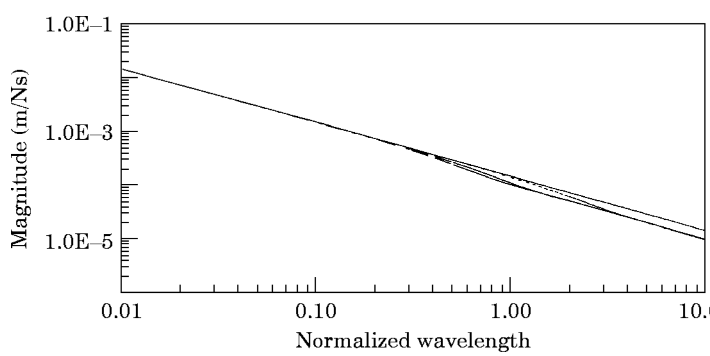

Figure 4. Receiver mobility magnitude for reference system. —, Y_r^{11}; · · · ·, Y_r^{12}; – – –, Y_r^{13}; – · · –, Y_r^{14}.

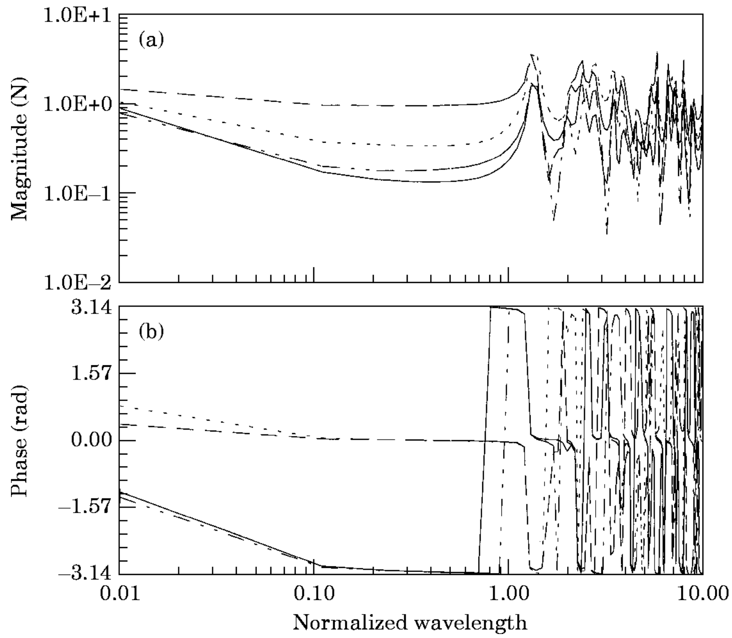

Figure 5. Magnitude (a) and (b) phase of forces in reference system. —, F^1; ---, F^2; – –, F^3; – · · –, F^4.

a single mass and the forces will have equal asymptotic values. Since there is no wave interaction at these frequencies the dynamics can be considered rigid body motion or mass controlled.

In the upper frequency region the wavelengths become small compared with the bays and wave interaction can occur. The rigid body motion is lost and the forces are dependent upon the beam sizes, materials, contact positions and boundary conditions. Both magnitude and phase are highly variable and no force consistently predominates. The response can be termed resonant.

The transition between rigid body and resonance behaviour occurs at the frequency of the fundamental system resonance. Approximately, this is when there is half a bending wavelength in each source bay indicating that the response of the receiver is having little effect upon the forces. The source can therefore be described as a constant force source. In general this cannot be assumed and the fundamental system resonance will be determined by both source and receiver structures.

With respect to the effective point mobility formulation the force ratios are of far greater interest than the forces. Shown therefore are the magnitude and corresponding phase of F^2/F^1, F^3/F^1 and F^4/F^1 in Figures 6(a) and (b). Again a clear distinction can be made between the rigid body and resonant regions.

The magnitudes of F^2/F^1 and F^3/F^1 in the rigid body region are both greater than unity whilst the ratio F^4/F^1 is close to unity. This observation is however largely trivial for, were the forces normalized by, for example, F^3, then all the ratios would be below unity.

It is interesting to observe that the footprint of the fundamental resonance is not seen in the force ratios and the character of the rigid body region has been extended. This is because the resonance is a global effect experienced by all of the forces and therefore not observed upon taking ratios.

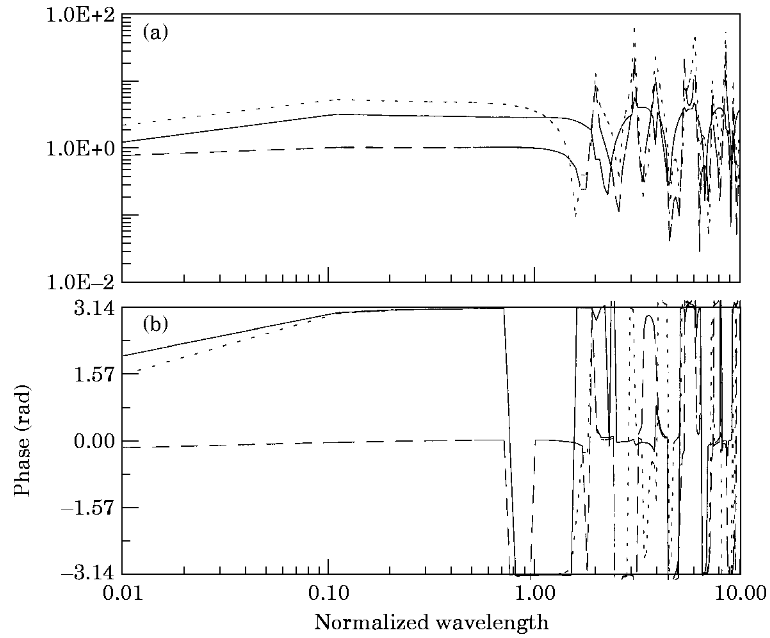

Figure 6. (a) Magnitude and (b) phase of force ratios in reference system. –, F^2/F^1; ---, F^3/F^1; – –, F^4/F^1.

In the rigid body region F^2/F^1 and F^3/F^1 are π out of phase with F^4/F^1: i.e., F^2 and F^3 move contrary to F^1 and F^4. The response can be considered approximately symmetrical about the system centre.

In the resonance region the magnitudes of the force ratios exhibit variations about unity within approximate bounds of $10^{\pm 2}$ which, for a conventional dB scale, equates to ± 40 dB. This corresponds to that shown experimentally by Mondot [8]. The phases can be considered random.

4. TRANSMITTED POWER ESTIMATES

By using the true force ratios the correct solution for the total transmitted power was obtained. This was used to normalize estimates obtained via the simple force ratio assumptions introduced by equations (8)–(10). Since the boundary conditions imposed do not permit moment induced power, any discrepancies between the correct and estimate solutions result only from differences between the true and the estimate translational force ratios. Each of the three simple force ratio estimates will be considered separately. It was noted that at certain frequencies (about 5%) the estimates produced negative transmitted power. At these frequencies the absolute value is used instead.

Shown in Figure 7 is the normalized estimate of the transmitted power, Q est U, with unit magnitude and zero phase assumed for the force ratios: i.e., equation (8). At the lower frequencies the estimate is accurate whilst at the high frequencies significant discrepancies occur. Although at a few frequencies the discrepancies are large, within a range of approximately $10^{\pm 3}$ (of ± 30 dB), the trend is unity.

Interestingly the transition from the lower to the upper frequency region is not according

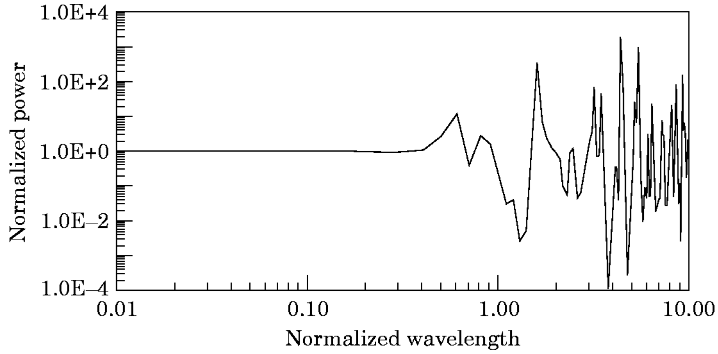

Figure 7. Normalized Q est U for reference system.

to the force ratios, but the source mobilities. It is explained by considering the magnitude of the effective point mobility, $|Y^{nn\Sigma}|$, where

$$
\begin{aligned}
|Y^{nn\Sigma}| = {} & \sum_{m=1}^{4} |Y^{1m}| \left|\frac{F^m}{F^1}\right| + \sum_{m=2}^{4} 2|Y^{11}||Y^{1m}| \left|\frac{F^m}{F^1}\right| \{\cos(\theta_{Y^{11}} - \theta_{Y^{1m}}) \cos\theta_{F^{m1}} \\
& + \sin(\theta_{Y^{11}} - \theta_{Y^{1m}}) \sin\theta_{F^{m1}}\} \\
& + \sum_{m=3}^{4} 2|Y^{12}||Y^{1m}| \left|\frac{F^2}{F^1}\right| \left|\frac{F^m}{F^1}\right| \{\cos(\theta_{Y^{12}} - \theta_{Y^{1m}}) \cos(\theta_{F^{m1}} + \theta_{F^{21}}) \\
& + \sin(\theta_{Y^{12}} - \theta_{Y^{1m}}) \sin(\theta_{F^{m1}} + \theta_{F^{21}})\} \\
& + 2|Y^{13}||Y^{14}| \left|\frac{F^3}{F^1}\right| \left|\frac{F^4}{F^1}\right| \{\cos(\theta_{Y^{13}} - \theta_{Y^{14}}) \cos(\theta_{F^{41}} + \theta_{F^{31}}) \\
& + \sin(\theta_{Y^{13}} - \theta_{Y^{14}}) \sin(\theta_{F^{41}} + \theta_{F^{41}})\}.
\end{aligned}
\tag{16}
$$

The influence of a force ratio is dependant upon its phase relationship with both the mobilities and the other force ratios. All such relationships have the form

$$f(\theta_Y, \theta_F) = \cos\theta_Y \cos\theta_F + \sin\theta_Y \sin\theta_F, \tag{17}$$

where θ_Y is the phase difference between two mobilities and θ_F the phase difference between two force ratios. The fundamental resonance of the system is not seen in the force ratios and their phase therefore is frequency invarient at, and just beyond, this frequency. Hence, the transition in the effective point mobility from the smooth lower region to the variable upper frequency region follows the phase of the mobilities: i.e., at the division between the mass and resonance controlled regions of the source mobilities.

The normalized estimate Q est P obtained when assuming the effective point mobility reduces to the point mobility, equation (9), is shown in Figure 8. Again the estimate is accurate in the mass controlled region large whilst discrepancies occur in the resonance controlled region. Further, the discrepancies are again within approximate limits of ± 30 dB with a trend to unity.

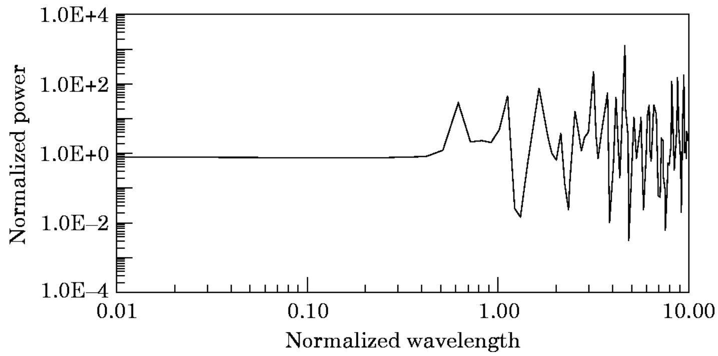

Figure 8. Normalized Q est P for reference system.

The normalized estimate Q est V obtained when assuming that the forces are given by the quotient of the free velocity at the contact point and the point mobility of the source at the point, equation (10) is shown in Figure 9. Again, the estimate is seen to be accurate in the mass region but variable in the resonance region; once more though the trend of the estimate is that of the true value with the discrepancies within the range of $10^{\pm 3}$.

For the simple system considered the three force ratio assumptions yield accurate estimates of transmitted power in the mass controlled region. In the resonance region, however, all three assumptions introduce discrepancies within a range ± 30 dB. In this region, the trend of each estimate does though seem to follow that of the true value.

The model is now used to highlight features expected in real installations i.e., the influence of the connector positions, the position of the excitation, the structural loss factor, the receiver structure and the number of connectors can be considered. To prevent the analysis from being specific to a particular system and to allow it to be more general a full discussion is deferred until these other systems have been considered.

4.1. INFLUENCE OF CONNECTOR POSITIONS

The results from three other configurations of connector positions were obtained. Two of these were with non-symmetric positions and one with the connectors placed symmetrically around a centrally positioned excitation force.

Typical normalized estimates of the transmitted power, Q est U, Q est P and Q est V, are shown in Figures 10–12. For all three cases the form of the mobility and force ratios

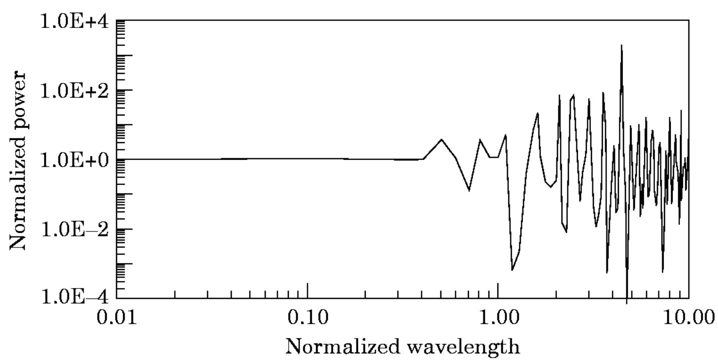

Figure 9. Normalized Q est V for reference system.

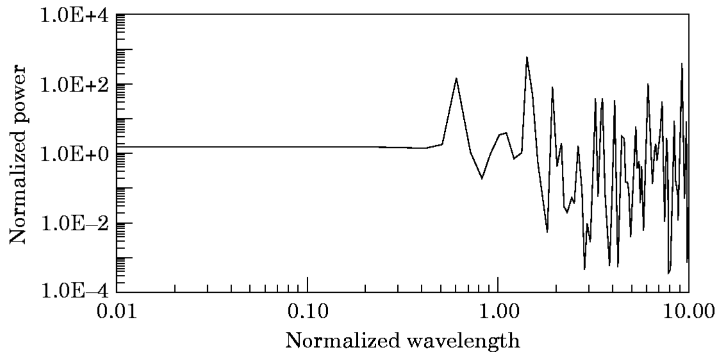

Figure 10. Normalized Q est U for connector set-up 1.

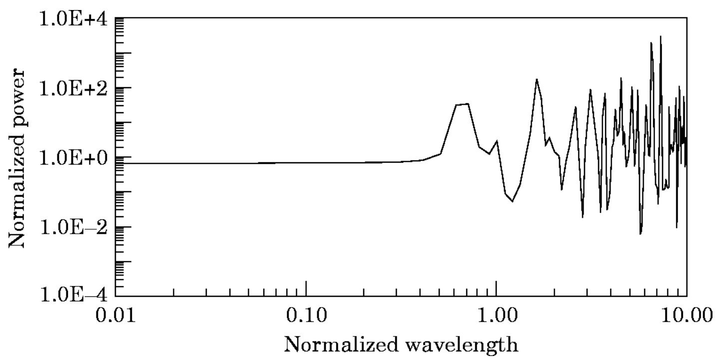

Figure 11. Normalized Q est P for connector set-up 2.

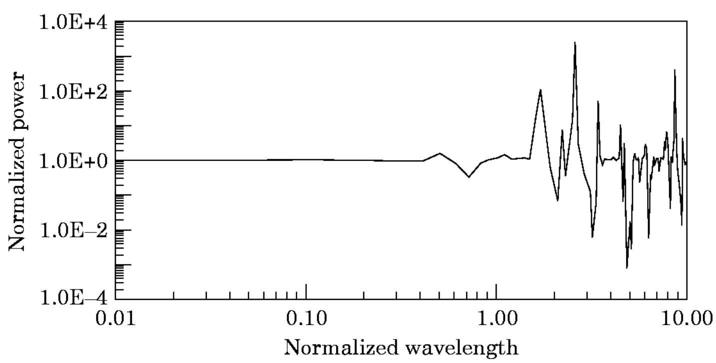

Figure 12. Normalized Q est V for connector set-up 3.

was similar to that seen in the reference system: i.e., they exhibited both mass and resonance controlled regions.

In the resonance region, the three estimates for all three systems introduce discrepancies similar in form to those seen for the reference system although with a different spectrum: i.e., except for a few discrete frequencies the estimate is within ± 30 dB of the true value.

In the mass region Q est P introduced a small discrepancy for all three systems. The estimate Q est U however only introduced a difference in one case (connector set-up 1) whilst Q est V was accurate for all three cases.

The results suggest that neither of the three simple force ratio assumptions can be relied upon in the resonance region. In the mass region Q est V seems to be the most reliable.

4.2. INFLUENCE OF EXCITATION POSITION

Two systems were considered where the position of the excitation force could be varied. One where the excitation force was in the end bay and one in which a force was positioned in each bay; here, all the forces have unit magnitude and all are in phase. For both systems the connector positions were as for the reference system.

For the end bay excitation the force ratio magnitude are shown in Figure 13(a) and the corresponding phases in Figure 13(b). The form of the ratios is familiar: i.e., a mass controlled region and a resonance region. In the mass region F^1 and F^2 are in phase, F^1 and F^3 are in phase and F^1 and F^4 are π out of phase. Unlike the reference system the response in the region is therefore not symmetrical about the centre.

In the resonance region, all three estimates of transmitted power introduce discrepancies similar to those seen previously; Q est U is shown in Figure 14. In the mass region however, the discrepancies for Q est U and Q est P were 6 dB and 3 dB respectively. These are larger than previously observed.

For the force per bay excitation, the magnitudes and phases of the force ratios can be expected to be similar in form to all those seen previously. It is noted though that in the mass region, all the forces will be in phase (the system simply moves in a translatory mode) and the response approximately symmetrical about the centre.

The normalized power estimate Q est P is shown in Figure 15. In the mass region the estimate is accurate whilst in the resonant region discrepancies familiar in form to those

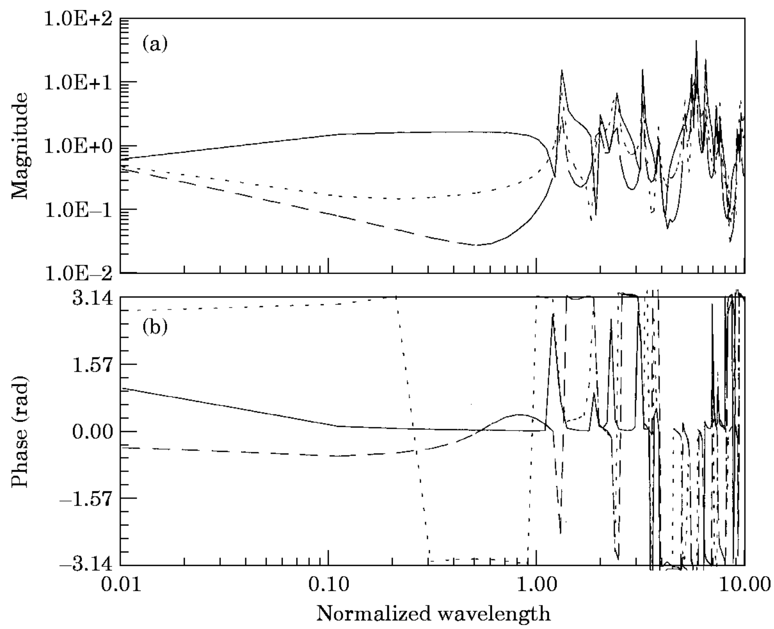

Figure 13. Magnitude (a) and phase difference (b) of force ratios for end bay excitation. –, F^2/F^1; ---, F^3/F^1; – –, F^4/F^1.

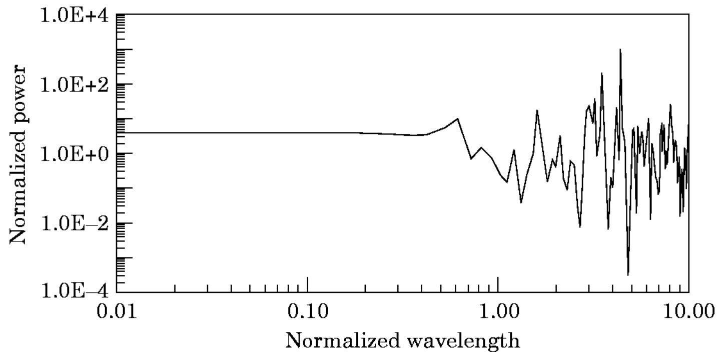

Figure 14. Normalized Q est U for end bay excitation.

seen for the other four-point systems are introduced. The results for Q est U and Q est V were similar.

4.3. INFLUENCE OF LOSS FACTOR

The effect of the structural loss factor is considered by increasing its value in the reference system from 0·001 (assumed in the system so far) to 0·01. As regards the source mobilities, it can be expected [3] that the increased loss factor will decrease the magnitude of the peaks in the resonance region but have no effect in the mass region.

The magnitudes of the forthcoming force ratios F^2/F^1, F^3/F^1 and F^4/F^1 are shown in Figure 16. The peaks in the general region have been reduced compared to those of the reference system.

The estimate Q est V is shown in Figure 17. As a direct consequence of the changes in mobility, the discrepancies in the resonant region are reduced, cf., Figure 9, and particularly so at the upper frequencies. The position in frequency of the discrepancies is however not changed. In the mass region the estimates are as accurate as before. Similar results were forthcoming for both Q est P and Q est U.

4.4. INFLUENCE OF RECEIVER

To assess the influence of the receiver structure the thickness of the receiver beam was reduced to that of the source beam.

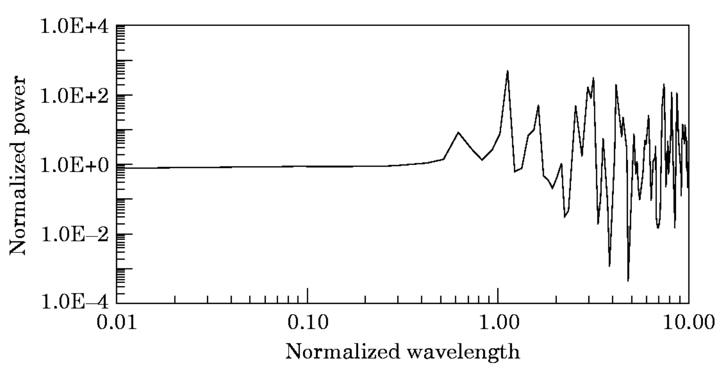

Figure 15. Normalized Q est P for force per bay excitation.

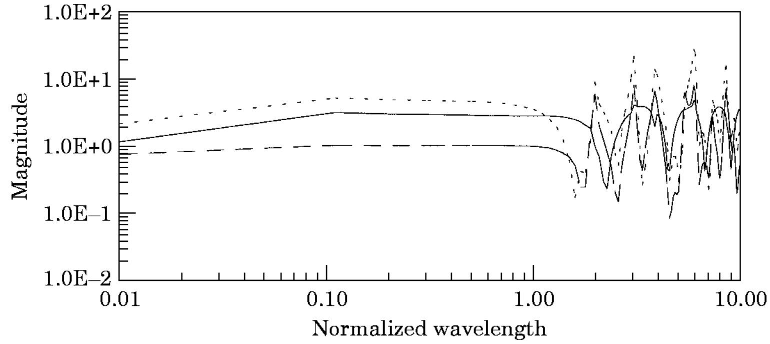

Figure 16. Magnitude of force ratios for 0·01 loss factor. –, F^2/F^1; ---, F^3/F^1; – –, F^4/F^1.

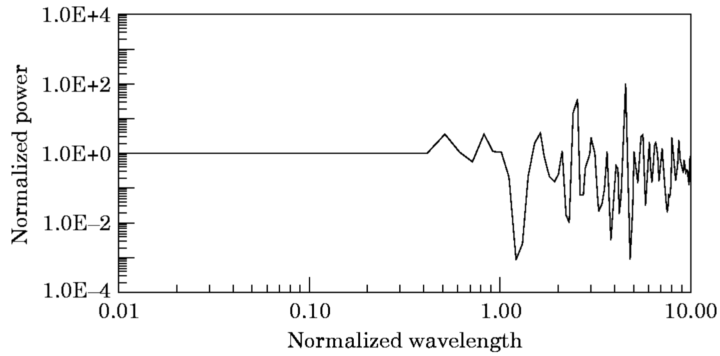

Figure 17. Normalized Q est V for 0·01 loss factor.

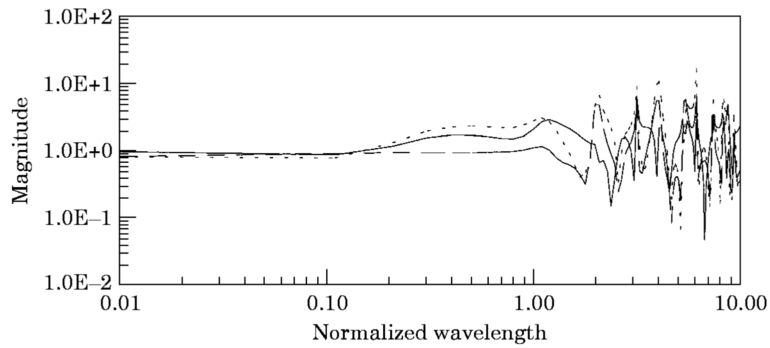

Figure 18. Magnitude of force ratios for 5 mm receiver. –, F^2/F^1; ---, F^3/F^1; – –, F^4/F^1.

The ensuing force ratio magnitudes for a 5 mm receiver beam are shown in Figure 18. These are clearly different to those of the reference system (Figure 6) revealing that the receiver structure is influential. Their form however is likewise with the other systems: i.e., a mass region and a resonance region.

In the mass region the accuracy of all the transmitted power estimates was similar to that seen for the reference system. In the resonant region however the discrepancies for Q est P and Q est U were reduced such that for many frequencies the discrepancies fell within the range ± 15 dB; Figure 19 shows Q est U (compare with Figure 7).

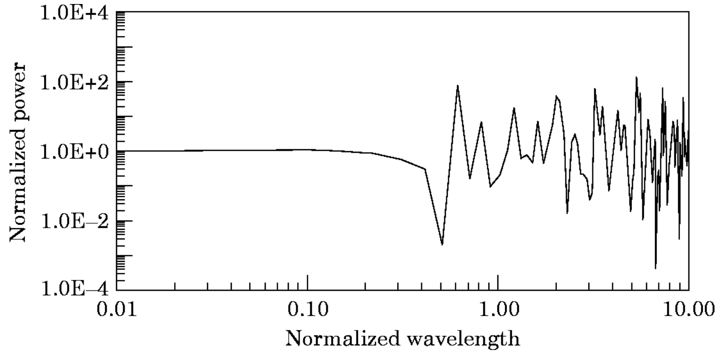

Figure 19. Normalized Q est U for 5 mm receiver.

4.5. INFLUENCE OF THE NUMBER OF CONNECTORS

To investigate whether the number of connectors influences the accuracy of the power estimates a system with eight connectors was modelled. As for the reference system, the connectors were positioned at random (though with a small standard deviation) and the excitation was maintained as a single force in the central bay. The thickness of the source and receiver beams were again 5 mm and 17 mm respectively and their material properties as for the reference system.

The magnitudes of the force ratios associated with the effective point mobility of the first connector are shown in Figure 20(a) and the corresponding phases in Figure 20(b). Once again both a mass controlled and a resonant region are observed. In the mass region the phase of the force ratios indicates that F^1, F^8 are π out of phase with F^2, F^7 and that F^4, F^5 are π are out phase with F^3, F^6. This indicates that likewise the reference system the

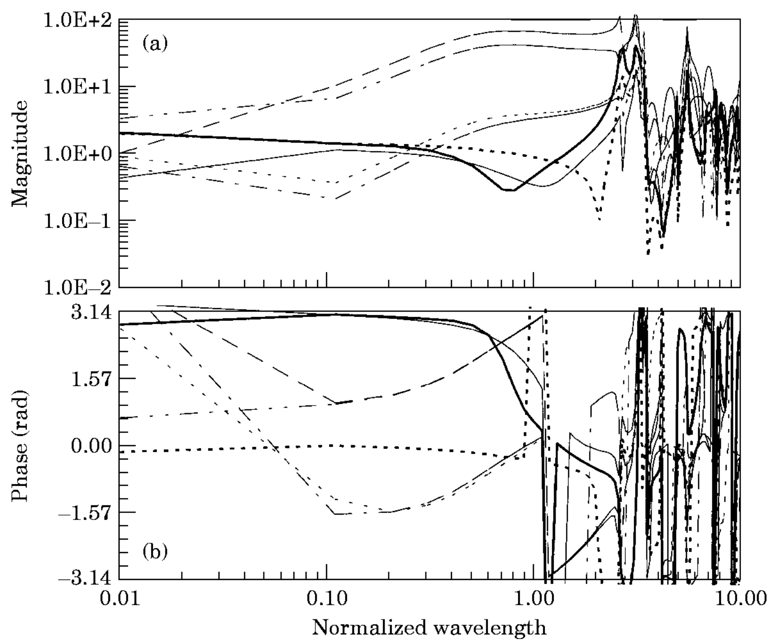

Figure 20. Magnitude (a) and phase (b) of force ratios for eight connectors. –, F^2/F^1; ---, F^3/F^1; – –, F^4/F^1; – · · –, F^5/F^1; – · –, F^6/F^1; —, F^7/F^1; - - - -, F^8/F^1.

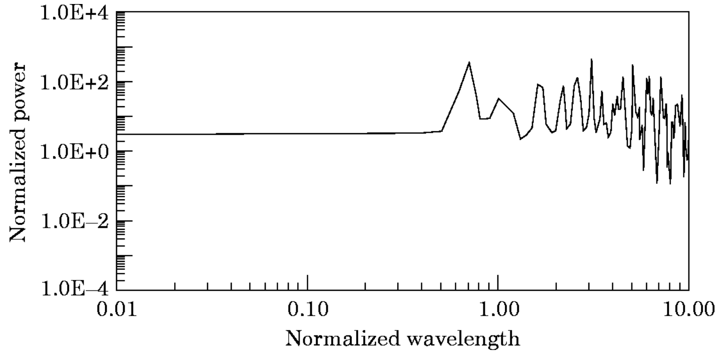

Figure 21. Normalized Q est P for eight connectors.

response is approximately symmetrical. In the resonance region, whilst the force ratios are again within approximate limits of ± 30 dB their trend has increased from unity to 3 dB.

In the mass region both Q est U and Q est V were accurate whilst Q est P overestimated by about 5 dB; the normalized power estimate Q est P is shown in Figure 21. In the resonance region the discrepancies of Q est U and Q est V had a similar form to those for the other systems. There is however a tendency for Q est P to overestimate in the resonant region.

5. DISCUSSION

Unfortunately, the amount of data involved in the calculations does not permit a mathematical discussion. Consider for example the full formulation for the total active power in a constant force source system of four contact points:

$$Q^{tot} = \mathrm{Re}\left\{\frac{(V_{sf}^1)^2}{|Y_s^{11\Sigma}|^2}\,Y_r^{11\Sigma} + \frac{(V_{sf}^2)^2}{|Y_s^{22\Sigma}|^2}\,Y_r^{22\Sigma} + \frac{(V_{sf}^3)^2}{|Y_s^{33\Sigma}|^2}\,Y_r^{33\Sigma} + \frac{(V_{sf}^4)^2}{|Y_s^{44\Sigma}|^2}\,Y_r^{44\Sigma}\right\}. \tag{18}$$

To allow the powers through the individual connectors to be compared, a common denominator would need to be introduced; i.e.,

$$Q^{tot} = \mathrm{Re}\left\{\frac{(V_{sf}^1)^2 Y_r^{11\Sigma}|Y_s^{22\Sigma}||Y_s^{33\Sigma}||Y_s^{44\Sigma}| + (V_{sf}^2)^2 Y_r^{22\Sigma}|Y_s^{11\Sigma}||Y_s^{33\Sigma}||Y_s^{44\Sigma}| + \cdots}{(|Y_s^{11\Sigma}||Y_s^{22\Sigma}||Y_s^{33\Sigma}||Y_s^{44\Sigma}|)^2}\right\}. \tag{19}$$

Upon remembering that, excluding the effect of different excitation components, each of the effective point mobilities has the form

$$Y_{ii}^{nn\Sigma} = Y_{ii}^{nn} + \sum_{m=1, m\neq n}^{N} Y_{ii}^{nm}\frac{F_i^m}{F_i^n}, \tag{20}$$

it is clear that the number of terms involved makes it impractical to multiply out the brackets in the equation. The common denominator would for example involve 256^2 terms. Subsequently the influence of a particular force ratio or mobility function cannot be assessed analytically.

Observation suggests that the position of the internal excitation is of most significance. This affects the response of the system and in particular whether in the mass controlled region it is symmetric or not. If the response is symmetric the accuracy in the mass controlled region of all three estimates—particularly Q est V—is good. For a non-symmetric response however the estimates in the region are, though less so for Q est V, poor. With respect to the force ratios the significant difference between the symmetric and the non-symmetric system is not that the assumed force ratios in the symmetric system approximate the true values better but rather that the ratios can be paired together. For the non-symmetric system they cannot. Because the mobility functions are independent of the excitation the influential factor must be the symmetrical properties of the free velocities. This suggests that from free velocity data an engineer could assess the suitability or otherwise of invoking for the mass controlled region one of the simple force ratio assumptions. In the resonance region the symmetry of the response does, though, have little effect upon the accuracy of the estimates of the power and all are poor.

When the connector positions are altered, though the discrepancies in the resonance region vary with frequency the trend is unaffected. In the mass region there is no significant effect. For practical purposes it is suggested therefore that providing the symmetrical properties of the system response is not dramatically altered, changing the contact positions has little influence upon the accuracy of the power estimates.

In the mass region, varying the loss factor has no effect upon the accuracy of the power estimates. In the resonance region, however, the discrepancies reduce as the loss factor is increased. This suggests a correlation between loss factor and overall accuracy. If a correlation were qualified and quantified an engineer could, prior to assembly, obtain the loss factor from mobility data and assess the expected overall accuracy in the resonance region of a power estimate based upon one of the three simple force ratio assumptions.

When the mobility of the receiver is increased towards that of the source the receiver becomes increasingly influential and its presence becomes manifested in the force ratios. The principal effect is to alter the frequency and magnitude of the resonance peaks. Though in the initial study a decrease in the magnitude of the peaks was seen, the complexities of the force ratios are such that this cannot be assumed to be a general result. Likewise, although the discrepancies between the true transmitted power and the estimates were seen to decrease, the complexities of the calculations are such that this too is not suggested to be a general result.

Finally, increasing the number of connectors is seen to have little effect upon the overall accuracy of the power estimates. This is interesting for it suggests that the discrepancies are, in general terms, independent of the number of connectors.

6. CONCLUDING REMARKS

An initial investigation has been undertaken to examine the viability of using simple force ratio assumptions to obtain—via the effective point mobility concept—estimates for the total transmitted power in a multi-point-connected system. The system considered was one-dimensional with a finite source and an infinite receiver and the behaviour therefore highly modal.

For the three simple force ratio assumptions, three estimates were obtaind for a reference system and the effects of the position of the connectors, the type of excitation, the loss factor, the receiver and the number of connectors upon these estimates were studied.

The results suggest that, when the system is mass controlled and approximates a symmetrical response, accurate estimates can be obtained. When the response is resonance controlled, however, all three estimates introduce large discrepancies. A significant reduction in these is achieved only by increasing the loss factor in the system.

A more detailed study is needed to interpret the results with respect to (i) source characterization and (ii) application of the effective point mobility to other systems.

For the effective point mobility formulation to become a realizable concept a greater understanding of the product of force ratio and mobility is required. To do this, one of two approaches can be taken. Either the product itself can be considered or else the mobility and force ratio can be considered separately. The first approach can be rejected since there are limits to the understanding obtained. With the second approach a distinction is made, however, between the contribution of the structure (the mobility) and the activity of the source (which contributes to the force). This provides a greater opportunity for understanding.

An investigation based upon the second approach has been undertaken [11]. This work is first concentrated upon generalizing structural characteristics and source activity and then, through a novel approach, the results are applied to obtain generalized statistical distributions for the force ratios. By using these distributions, statistical distributions for overall source descriptor functions are then obtained. The work is to be reported in later papers.

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